# DECISION ANALYSIS AND VALIDATION OF VALUE FOCUSED THINKING DECISION MODELS USING MULTIVARIATE ANALYSIS TECHNIQUES

THESIS

James L. Pruitt, Captain, USAF

AFIT-OR-MS-ENS-11-17

**DEPARTMENT OF THE AIR FORCE**
**AIR UNIVERSITY**
**AIR FORCE INSTITUTE OF TECHNOLOGY**

**Wright-Patterson Air Force Base, Ohio**

AFIT-OR-MS-ENS-11-17

# DECISION ANALYSIS AND VALIDATION OF VALUE FOCUSED THINKING DECISION MODELS USING MULTIVARIATE TECHNIQUES

THESIS

Presented to the Faculty

Department of Operational Sciences

Graduate School of Engineering and Management

Air Force Institute of Technology

Air University

Air Education and Training Command

In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Operations Research

James L. Pruitt, BS

Captain, USAF

March 2010

AFIT-OR-MS-ENS-11-17

DECISION ANALYSIS AND VALIDATION OF VALUE FOCUSED THINKING DECISION MODELS USING MULTIVARIATE TECHNIQUES

James L. Pruitt, BS
Captain, USAF

Approved:

__________//Signed//___________________ 9 March 2011 ___
Dr. Kenneth Bauer (Co-Chairman) date

__________//Signed//___________________ 9 March 2011 ___
Dr. Jeffery Weir (Co-Chairman) date

AFIT-OR-MS-ENS-11-17


## Abstract

Decision Analysis (DA) is a useful tool to assist decision makers (DM) with difficult and complex decisions using mathematical models. Value Focused Thinking (VFT) models are a useful DA tool widely employed in the Air Force. However, VFT models are rarely validated.

This research will attempt to validate any given VFT model and provide insight into the discriminating attributes of the alternative set. First, a two group discriminant analysis is applied the alternative set given the prior knowledge of the selected alternatives. Next, compromise programming is used attempt to minimize the distance between the posterior probability of an alternative being selected and its current weighted value by varying the weights. This set of optimized weights is then used in the two group discriminant analysis to classify the alternative set and attempt to validate the VFT model by selecting the same subset of alternatives chosen by the DM. Additionally, this process will provide insight into what attributes of a given alternative set are actually the discriminating factors in the decision which may or may not be the attributes that are most important to or most heavily weighted by the DM.

AFIT-OR-MS-ENS-11-17

## DEDICATION

*To Wife and Son*

## Acknowledgments

I would like to express my sincere appreciation to my faculty advisors, Dr. Kenneth Bauer and Dr. Jeffery Weir, for their direction, guidance and support throughout this thesis effort. I would also like to thank my wife for her support and willingness to proofread the many iterations of this document.

James L. Pruitt

## Table of Contents

## List of Tables

Page

VALIDATION OF VALUE FOCUSED THINKING DECISION MODELS USING MULTIVARIATE TECHNIQUES

# I. Introduction

Air Force decision makers (DM) are forced to make difficult decisions that can have significant impact to future force capabilities and structure. Often these decisions are made more difficult by the fact that the goals of the decision are often conflicting. Decision Analysis (DA), specific to this research Value Focused Thinking (VFT), provides a tool that allows analysts to help the DM make tradeoffs between conflicting goals. A DM needs to have a detailed, robust and mathematically rigorous model to aid in their decision making.

## I.A Background

Decision makers and analysts often employ VFT models to assist in making difficult decisions. The VFT methodology is well documented and often used in today's Air Force. The VFT process generates a VFT model that should encompass the values and objectives of the DM and any stakeholders. Once the VFT hierarchy is completed, attributes or measures are weighted; alternatives are scored, evaluated, and then ranked. Often a selection threshold is set and any alternatives with a value score from the VFT model greater than the threshold are selected. Instead of a value score threshold, a budget constraint may be applied. The top value score alternatives up to a resource or budget constraint would be selected in this case.

Alternatively, the VFT results may not reflect the final decision and some alternatives that scored poorly in the VFT model may end up being selected based upon other factors that were not or could not be reflected in the VFT model. In a case such as this, there may be a logical set of weights that can be applied to the VFT model to account for the DM's change in values to reflect the decision outcome. In this case, the DM may be weighting an attribute that he feels is important but due to the value scores of the alternative set the attribute has little discriminatory power. For example, if a set of alternatives all scored very well in the most important attribute to the DM, but varied greatly in an attribute that was less important, through discriminant analysis we can provide insight into whether that less important attribute is actually having the most influence in making the decision.

### I.B Problem Statement

Decision makers often have to make complex and difficult decisions. Ensuring decision analysis techniques such as VFT are accurately applied and provide valid, repeatable and accurate results helps to justify allocation of scarce resources to the many areas they are needed. This research provides a framework to help ensure consistent decision analysis techniques are used and to help provide insights as a result of the decision that may not be readily apparent.

### I.C Research Scope

This thesis will evaluate the validity of a given VFT model. We will address whether the VFT model accurately reflects the DM's decision process in light of the alternatives presented him using discriminant analysis with a two group problem. After this validation process we provide insight to the DM and stakeholders by determining which attributes of the alternatives had the most influence on whether they were selected or not selected.

**I.D Assumptions**

This research does not require any specific assumptions regarding the VFT model or the alternative set.

**I.E Thesis Organization**

The remainder of this thesis is composed of four chapters. Chapter 2 consists of a literature review of DA, VFT, weighting techniques, sensitivity analysis, multivariate analysis, and discriminant analysis techniques. Chapter 3 outlines the methodology used by this research including discriminant analysis modeling, and compromise programming optimization of the discriminant weights. Chapter 4 consists of the results of the application of the methodologies covered in chapter 3 when applied to alternative sets and VFT models. Finally, chapter 5 discusses the conclusions and suggests opportunities for future research.

# II. Literature Review

## II.A Introduction

The literature review for this work encompasses relevant material regarding Decision Analysis (DA) and value models. We will discuss the fundamentals of those disciplines along with some of the multivariate analysis techniques that are pertinent to this research.

## II.B Decision Analysis

Decision Analysis (DA) is an analysis tool that allows a Decision Maker (DM) to have a repeatable, mathematically rigorous process to aid in making difficult decisions. “DA provides structure and guidance for thinking systematically about hard decisions.” (Clemen & Reilly, 2001). As described by Clemen, we must start by identifying the DM or DMs. Next, the DA process begins by identifying the decision situation and ensuring we understand the objectives. We then identify the alternatives to be considered. Next, follows the decomposition and modelling of the problem. The VFT model will show us how the alternatives faired and we can choose the best alternative or if necessary try to generate new alternatives if those provided did not meet objectives to the levels required by the DM. We can also conduct sensitivity analysis on the results of the model and determine if further analysis is needed. Finally if no further analysis is required we can implement our chosen alternative. DA is not intended to replace the DM or make the decision for them. DA is a useful tool to provide insights and help the DM make an informed decision.

## II.C Value Focused Thinking

The methods derived in from this research can be applied to any Value Focused Thinking (VFT) model that is used to evaluate alternatives and then select some subset of the alternatives to be selected for funding or some other function. VFT techniques differ from alternative based techniques since the latter approach considers the best of the available alternatives (Keeney, 1992). A VFT approach shows how much value is provided by an alternative and can lead to development of new alternatives based upon revelations provided by the value model.

Problem identification is an important first step in the VFT process. It is essential to make sure you are looking at the right problem. Discussions with the DM and stakeholders provide clarity with regards to the problem identification. The DM's values are determined by what is important to him or her. It is sometimes useful to consider strategic plans and objectives that have been developed by the DM. They can usually provide insight into the DM's values and aid in development of better alternatives. Weights are assigned to the value attributes using one of various weighting techniques and based upon the importance of the value to the decision maker. The more important the attribute's value is to the DM, the more weight it will have in the model. Alternatives are then considered and scored based upon value functions for the individual value measures. The resulting VFT model output will give an overall value score for each alternative. This will provide insight into which alternative will give the most value based upon the DM's preferences. Sensitivity analysis would then be performed to check the robustness of the VFT model.

## II.D VFT Weighting Techniques

There are several weighting techniques available for use in VFT models and this research is applicable to any weighting technique. Pöyhönen describes some weighting techniques including Direct weighting, Simple Multiattribute Rating Technique (SMART), Swing weighting, Tradeoff weighting, and the Analytic Hierarchy Process (AHP) (2001).

In Direct weighting, the DM just assigns weights to each attribute. Typically the weights are assigned such that they sum to one. This is not necessary but it is usually easier for the DM and stakeholders to relate to the meaning of the weight if it is scaled on a zero to one scale. Assigning weights with the SMART technique involves ranking the importance of changes in attributes when an attribute moves from its lowest score to its highest score. Ratio estimates are assigned sequentially starting with the least important attribute. In SWING weighting the DM is given a hypothetical alternative that has the worst score for each attribute. He then chooses his most important attribute to increase to its highest level and assigns 100 points to it. The next most important attribute is then chosen to increase to its highest level and assigned something less than 100 points. This process continues until all attributes are assigned a weight. In the Tradeoff method the DM is asked to compare 2 hypothetical alternatives that differ in two attribute measures only with all other attribute scores held constant. The DM then compares the alternatives and is asked to change the attribute scores until he is indifferent between the two alternatives. That is, he likes both alternatives equally as well even though they have 2 attributes that are not the same. This would have to be done n-1 times to get all the indifference statements, where n is the number of attributes. The indifference statements are used to generate equations relating the respective weights and value functions for the attributes. Those n-1 equations plus a normalization equation allow calculation of the weights. In AHP the DM is asked to compare two attributes at a time and give a relative importance between the two for each combination of two weights. The relative importance is determined by a weight ratio assigned by the DM for each pair of attributes. The weight ratio is typically an integer from 1 to 9. Once this is accomplished for all pair combinations of attributes, the weights are usually derived from the principle eigenvector of the comparison matrix (Pöyhönen & Hämäläinen, 2001).

**II.E Sensitivity Analysis**

Often there is uncertainty in many key parts of the DA process and we need to conduct analysis on the results of the VFT model with regards to this uncertainty. Typically this is done by varying the weights. This allows us to look for changes in the rankings of the alternatives as the weights are varied. If the rankings change with small changes in the weights then the decision is said to be sensitive to changes in that weight or weights. If the rankings don't change or only change with proportionally large variation in the weights then the decision is insensitive to the weight or weights.

Clemen and Reilly discuss methods for one-way and two-way sensitivity analysis where one (or two) weights are varied while all other weights are kept proportionate to their original weights (Clemen & Reilly, 2001).

Bauer, Parnell, and Meyers present a method using Response Surface Methodology to perform higher order sensitivity analysis. The value functions from the

VFT model are transformed into a response function of the uncertain variables. RSM was used to create a sensitivity analysis framework that allowed simultaneous perturbation of a number of uncertain variables (Bauer et al, 1999).

Ringuest presents a methodology where the $L_1$ and $L_\infty$ metrics are minimized subject to linear constraints (Ringuest, 1997). He suggests that since the constraints are linear, solving the linear program minimizing the $L_1$ and $L_\infty$ metrics will completely specify the solutions which minimize the $L_p$ metric. The $L_p$ metric is the generalized form for P effect on the relative contribution of individual deviations (Ringuest, 1997).

## II.F Multivariate Analysis

Dillon and Goldstein define Multivariate analysis as the application of methods that deal with large numbers of measurements made on each object in one or more samples simultaneously (1984). The simultaneous aspect of the multiple variables and the analysis of all the variables at once instead of one-way analysis is the key point to multivariate analysis

## II.G Discriminant Analysis

Discriminant analysis is a statistical technique for classifying individuals or objects into mutually exclusive and exhaustive groups on the basis of a set of independent variables (Dillon & Goldstein, 1984). This is done by deriving a linear combination of the independent variables that determines the difference between the *a priori* defined groups so that misclassification is minimized. The technique attempts to maximize the between group variance relative to within group variance. The linear combination is determined by a weighted average of each object or individual's scores on the independent variables. This is then turned into *a posteriori* probability of being assigned to each group. How well the discriminant function performs is determined by how well the function classifies the objects or individuals.

### II.H Compromise Programming

Compromise programming is a specific form of multicriteria optimization programming that attempts to minimize the distance from an objective or ideal point to the alternative space. The general form is shown below.

Given a distance measure , the compromise programming problem is given by , where is the ideal point and any appropriate distance measure as a function of (Ehrgott, 2005). Typically, norms such as the norm or sums of square difference are used as the distance measure.

### II.I Summary

This chapter presents a review of literature that provided key elements of DA, VFT, weighting techniques, and sensitivity analysis. A vital part of this research centers on the Multivariate analysis technique of Discriminant Analysis. Uncertainty inherent to difficult decisions will allow us to explore application of Discriminant Analysis techniques to assess sensitivity of VFT models with regards to a given alternative set.

# III. Methodology

## III.A Introduction

The purpose of this chapter is to present the methods used to assess the robustness of a VFT model's grouping of selected and non-selected alternatives based on a given set of alternatives and global weights. This research will attempt to find the best set of global weights that will generate the same selection grouping of alternatives generated by the VFT model. First, this research uses discriminant analysis to determine if a discriminant function can reasonably classify the alternative set into the selected and non-selected groups. Next, the posterior probabilities generated by the discriminant analysis are used along with the value scores of the alternatives to create a math programming model to find the optimal set of weights that produce alternative value assessments that are as close as possible to the posterior probability estimates.

## III.B VFT Model

This research assumes that a given VFT model was properly formulated and in the form shown in equation 1, weightings were chosen by the DM or analysts in an appropriate fashion and the results of the VFT model were attained.

(1)

where:

: the evaluation measures for alternative *j* for each of the *i* attributes
: the weight associated with attribute *i*.
: the individual value functions associated with attribute *i*
: the overall value score of alternative *j*
N: the number of attributes, $i$ =1 to N
n: the number of alternatives, $j$=1 to n

The alternatives can then be ordered based upon their value score such that . Let **T** be a threshold of the value score that divides the alternatives into 2 sets. These sets represent the alternatives that were selected and rejected based on the threshold, **T**. It is possible that the sets are constructed in some other manner independent of the value model.

### III.C Discriminant Analysis

Discriminant analysis is "a statistical technique for classifying individuals or objects into mutually exclusive and exhaustive groups on the basis of a set of independent variables" (Dillon & Goldstein, 1984). Discriminant analysis tries to find a linear combination of the independent variables that will discriminate between the *a priori* defined groups while minimizing misclassification error. This is accomplished by maximizing the between-group variance relative to the within-group variance (Dillon & Goldstein, 1984).

Discriminant analysis uses a scoring system that assigns each object a score that is a weighted average of the object's values on the set of independent variables. This discriminant score is then transformed into an *a posteriori* probability that assigns a likelihood of the object belonging to each of the groups. In this research, the *a priori* groups are defined by VFT results and any supplemental analysis used by the DM. The independent variables used in the discriminant analysis are the value function scores of a given alternative for each attribute assessed in the VFT model. The idea is to model the alternatives evaluated in the VFT model as multivariate vectors of their individual value function assessments.

The equation used to generate the vector of discriminant weights using Fisher's Approach to a two group problem is shown in equation 2 below (Dillon & Goldstein, 1984):

(2)

where:

: the vector of discriminant weights
: the pooled sample variance-covariance matrix.
: the mean of the observations in the i$^{th}$ population

The discriminant scores are then created by equation 3 below.

(3)

where:

: the vector of discriminant scores
: the matrix of alternative value scores from VFT model where each row is an alternative and the columns are the attribute value scores
: the vector of discriminant weights

Once the discriminant scores are obtained, classification is made by comparing the discriminant score to a midpoint between the means of the two population groups. This midpoint is calculated using equation 4 below. A discriminant score greater than the midpoint would be assigned to group 1 and less than would be assigned to group 2 (Bauer K. W., 2010):

(4)

Calculation of the *a posteriori* probability is now necessary by using the following equation 5.

(5)

where:

: the *a posteriori* probability of alternative *j* being in the selected group

**III.D Compromise Programming**

Once the discriminant analysis has calculated the *a posteriori* probability of each alternative **x**, we use that posterior probability as the ideal point in a compromise programming problem that minimizes the difference between the value of the alternative and the posterior probability of being selected. The thought behind this is that an alternative that is selected should have a high VFT value score and also a high posterior probability of being in the selected population. Minimizing the difference between those numbers should increase the separation between the selected and non-selected populations. Equation 6 below shows the compromise programming problem with the posterior probability of the *j*th alternative being classified in the selected group as the ideal point.

(6)

where:

: the value of the *j*th alternative for the *i*th attribute where $i = 1,\ldots,N$ and $j = 1,\ldots,n$

: the weight of the *i*th attribute

### III.E Validation and Analysis

Once the new weight vector is obtained from the compromise programming, the classification can be tested using the two group discriminant analysis method. If the new discriminant does a reasonable job of classifying the alternatives relative to the original decision than we can conclude that the decision being made is statistically consistent and valid.

Given the new weights we may have some new insights into the alternative set. Any attribute weight that was reduced to zero or near zero was done so because it had no discriminatory power. Any attribute weights that are very large have significant discriminatory power. This will provide insight into which attributes actually determined which alternatives were selected and may or may not reflect the weights determined from the VFT process.

### III.F Summary

This chapter consisted of the methodology used in this research to validate VFT models and provide insight into a decision based upon a given alternative set. This research uses a given alternative set that has been processed through a VFT model and the selection outcome determined by a VFT value score threshold or some other method. Then, two group discriminant analysis techniques and compromise programming methods are applied. This method shows that statistically there is a set of weights that will produce the same selected and non-selected group and provide insights into what attributes of the alternatives are the discriminating factors. A copy of the MATLAB code utilized during this research is includes in Appendix A.

# IV. Results and Analysis

## IV.A Introduction

This chapter applies the methodologies discussed in chapter 3 to a VFT model and alternatives of an unspecified Air Force Major Command (MAJCOM), and to a JIEDDO VFT model and set of alternatives used previously in Willy's thesis (Willy, 2009). In each case we will validate the decision by using the discriminant analysis and compromise programming techniques identified in chapter 3 to duplicate the decision using statistical rigor. Then, for each alternatives set we will utilize the weights generated by the previous process to see if any insights are gained by the results. Finally, we will demonstrate an example of a poor hit rate from an inconsistent decision.

## IV.B MAJCOM Data

The set of MAJCOM alternatives is shown in Appendix B. The selection/non-selection cutoff thresholds used were a VFT value score of 0.5 and above for the selected population, 0.6 and above, 0.7 and above, and finally 0.8 and above. Initially the weights were only allowed to vary by ±10% and ±50% of their original value. This yielded weights as shown in Table 1 and Table 2 below.

Table 1 MAJCOM Weights for ±10% Bounds

| ±10% of Original Weights | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Selection | Attribute Weights | | | | | | | | | | | | | |
| Threshold (# Sel / | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 0.6 (30/42) | 0.066 | 0.16 | 0.088 | 0.187 | 0.153 | 0.0108 | 0.033 | 0.0162 | 0.066 | 0.0724 | 0.0738 | 0.0296 | 0.0221 | 0.0221 |
| 0.7 (25/42) | 0.066 | 0.16 | 0.088 | 0.187 | 0.153 | 0.0108 | 0.033 | 0.0162 | 0.066 | 0.0724 | 0.0738 | 0.0296 | 0.0221 | 0.0221 |
| 0.8 (12/42) | 0.066 | 0.16 | 0.088 | 0.187 | 0.153 | 0.0108 | 0.033 | 0.0162 | 0.066 | 0.0724 | 0.0738 | 0.0296 | 0.0221 | 0.0221 |
| Original Weights | 0.06 | 0.17 | 0.08 | 0.17 | 0.17 | 0.012 | 0.03 | 0.018 | 0.06 | 0.06578 | 0.08211 | 0.03289 | 0.02461 | 0.02461 |
| | (yellow) | Denotes weight set to Lower bound | | | (green) | Denotes weight set to Upper bound | | | | | | | | |

Table 2 MAJCOM Weights for ±50% Bounds

| ±50% of Original Weights | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Selection | Attribute Weights | | | | | | | | | | | | | |
| Threshold (# Sel / | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 0.6 (30/42) | 0.03 | 0.085 | 0.12 | 0.255 | 0.216 | 0.006 | 0.045 | 0.027 | 0.09 | 0.0434 | 0.0411 | 0.0165 | 0.0123 | 0.0123 |
| 0.7 (25/42) | 0.03 | 0.085 | 0.12 | 0.255 | 0.255 | 0.008 | 0.015 | 0.027 | 0.09 | 0.0329 | 0.0411 | 0.0165 | 0.0123 | 0.0123 |
| 0.8 (12/42) | 0.03 | 0.085 | 0.12 | 0.255 | 0.1262 | 0.018 | 0.045 | 0.027 | 0.09 | 0.0329 | 0.0477 | 0.0493 | 0.03692 | 0.03692 |
| Original Weights | 0.06 | 0.17 | 0.08 | 0.17 | 0.17 | 0.012 | 0.03 | 0.018 | 0.06 | 0.06578 | 0.08211 | 0.03289 | 0.02461 | 0.02461 |
| | | Denotes weight set to Lower bound | | | | | Denotes weight set to Upper bound | | | | | | | |

After looking at the results and noticing that several statistical weights were consistently set to their upper and lower bounds, we changed the bounds to allow the weights to vary from 0 to 1. This provided some interesting results shown in Table 3 below.

Table 3 MAJCOM Weights for 0 to 1 Bounds

| 0 to 1 Bound for Weights | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Selection | Attribute Weights | | | | | | | | | | | | | |
| Threshold (# Sel / | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 0.6 (30/42) | 0.085 | 0 | 0.136 | 0.423 | 0.129 | 0 | 0.026 | 0.111 | 0.082 | 0.008 | 0 | 0 | 0 | 0 |
| 0.7 (25/42) | 0 | 0 | 0.087 | 0.322 | 0.103 | 0 | 0 | 0.253 | 0.206 | 0.029 | 0 | 0 | 0 | 0 |
| 0.8 (12/42) | 0 | 0 | 0.101 | 0.017 | 0 | 0 | 0.007 | 0 | 0.467 | 0 | 0 | 0.408 | 0 | 0 |
| Original Weights | 0.06 | 0.17 | 0.08 | 0.17 | 0.17 | 0.012 | 0.03 | 0.018 | 0.06 | 0.06578 | 0.08211 | 0.03289 | 0.02461 | 0.02461 |

As we can see, there were several statistical weights set to zero and significant portions of the weight assigned to only a few attributes. The hit rates, shown in Table 4, were excellent for the resultant discriminant analysis. This statistically validates the MAJCOM VFT model. We were able to use a statistical method in discriminant analysis to verify that the selection grouping developed from the VFT model were consistent and mathematically sound.

Table 4 MAJCOM Confusion Matrices

| MAJCOM DATA ±10% Bound | | | |
|---|---|---|---|
| 0.6 Threshold (30/42) | | Select | Non Select |
| | Select | 30 | 0 |
| | Non Select | 0 | 12 |
| | | Hit Rate | 100.0% |
| 0.7 Threshold (25/42) | | Select | Non Select |
| | Select | 25 | 0 |
| | Non Select | 0 | 17 |
| | | Hit Rate | 100.0% |
| 0.8 Threshold (12/42) | | Select | Non Select |
| | Select | 12 | 0 |
| | Non Select | 1 | 29 |
| | | Hit Rate | 97.6% |

| MAJCOM DATA ±50% Bound | | | |
|---|---|---|---|
| 0.6 Threshold (30/42) | | Select | Non Select |
| | Select | 30 | 0 |
| | Non Select | 0 | 12 |
| | | Hit Rate | 100.0% |
| 0.7 Threshold (25/42) | | Select | Non Select |
| | Select | 25 | 0 |
| | Non Select | 0 | 17 |
| | | Hit Rate | 100.0% |
| 0.8 Threshold (12/42) | | Select | Non Select |
| | Select | 12 | 0 |
| | Non Select | 1 | 29 |
| | | Hit Rate | 97.6% |

| MAJCOM DATA 0 to 1 Bound | | | |
|---|---|---|---|
| 0.6 Threshold (30/42) | | Select | Non Select |
| | Select | 30 | 0 |
| | Non Select | 0 | 12 |
| | | Hit Rate | 100.0% |
| 0.7 Threshold (25/42) | | Select | Non Select |
| | Select | 25 | 0 |
| | Non Select | 0 | 17 |
| | | Hit Rate | 100.0% |
| 0.8 Threshold (12/42) | | Select | Non Select |
| | Select | 12 | 0 |
| | Non Select | 1 | 29 |
| | | Hit Rate | 97.6% |

This statistically validates the MAJCOM VFT model. We were able to use a statistical method in discriminant analysis to verify that the selection grouping developed from the VFT model were consistent and mathematically sound.

In trying to determine how the discriminant function and the compromise programming are generating the statistical weights it is useful to consider the mean of the selected group when compared to the mean of the non-selected group and the variance of the two groups. Table 5 shows a portion of this data with the full results in Appendix C. This provides insight into what attribute measures are actually influencing the decision. Those measures with statistical weight assigned contribute to explaining how the alternatives were assigned to the selected and non-selected groups and the measures that were assigned zero weight either were not a discriminating factor or had too much in group variation to be useful.

Table 5 MAJCOM Sample of Means and Variances

| 0.6 Selection Cutoff | Measure 1 | Measure 2 | Measure 3 | Measure 4 | Measure 5 | Measure 6 |
|---|---|---|---|---|---|---|
| Selected Mean | 0.950 | 0.847 | 0.610 | 0.831 | 0.780 | 0.467 |
| Non-selected Mean | 0.813 | 0.729 | 0.125 | 0.342 | 0.575 | 0.500 |
| Selected Variance | 0.015 | 0.019 | 0.180 | 0.070 | 0.024 | 0.257 |
| Non-selected Variance | 0.092 | 0.069 | 0.051 | 0.159 | 0.091 | 0.273 |

| 0.7 Selection Cutoff | Measure 1 | Measure 2 | Measure 3 | Measure 4 | Measure 5 | Measure 6 |
|---|---|---|---|---|---|---|
| Selected Mean | 0.970 | 0.860 | 0.628 | 0.847 | 0.808 | 0.520 |
| Non-selected Mean | 0.824 | 0.744 | 0.241 | 0.461 | 0.594 | 0.412 |
| Selected Variance | 0.012 | 0.019 | 0.180 | 0.079 | 0.020 | 0.260 |
| Non-selected Variance | 0.068 | 0.052 | 0.121 | 0.151 | 0.070 | 0.257 |

| | | | |
|---|---|---|---|
| | Denotes a weighted attribute with 0 to 1 bound | | Denotes attribute with zero weight assigned |

When the bounds for the weights are relaxed to 0 to 1, weight is assigned to those measures that typically have the greatest difference between selected and non-selected means. There were some instances where there was a significant difference in means but a zero weight was still assigned. This usually occurred when the variances of the two groups was large enough that it reduced the effectiveness of the differences in the means to discriminate.

### IV.C JIEDDO Data

The same techniques were applied to a set of alternative data shown in Appendix D and value model from JIEDDO proposals (Willy, 2009). In the original VFT model there were actually 13 measures. However, the given alternative set scored exactly the same value for the Training Level measure. This measure was removed for the analysis portion since it had no discriminatory information. Runs were conducted using value scores of 0.4 and above for the selected group, 0.5 and above, 0.6 and above, and finally the actual selected group decided by the JIEDDO DMs, which did not follow the VFT model ranking exactly.

The results were similar to the MAJCOM data in that there were only a few attributes that were weighed heavily and some others were set to zero as shown in Table 6 below.

Table 6 JIEDDO Weights for 0 to 1 Bounds

| Selection | Attribute Weights | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Threshold (# Sel / | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 0.4 (25/30) | 0 | 0.076 | 0 | 0 | 0 | 0 | 0 | 0 | 0.001 | 0.923 | 0 | 0 |
| 0.5 (19/30) | 0 | 0.189 | 0 | 0.171 | 0 | 0 | 0.393 | 0 | 0 | 0.247 | 0 | 0 |
| 0.6 (7/30) | 0 | 0.123 | 0 | 0.49 | 0.051 | 0 | 0 | 0.234 | 0 | 0 | 0.102 | 0 |
| JIEDDO Selection | 0.041 | 0.398 | 0 | 0 | 0 | 0 | 0.134 | 0.377 | 0.05 | 0 | 0 | 0 |
| Original Weights | 0.056 | 0.176 | 0.056 | 0.112 | 0.11 | 0.056 | 0.091 | 0.037 | 0.056 | 0.1 | 0.087 | 0.05 |

Similar to the MAJCOM data results, weight was distributed across only a few of the attributes. Even with such a weight distribution we still had an excellent classification hit rate as shown in Table 7. This statistically validates the JIEDDO VFT model. We were able to use a statistical method in discriminant analysis to verify that the selection grouping developed from the VFT model were consistent and mathematically sound.

Table 7 JIEDDO Confusion Matrices

| JIEDDO DATA ±10% Bound | | | |
|---|---|---|---|
| 0.4 Threshold (25/30) | | Select | Non Select |
| | Select | 25 | 0 |
| | Non Select | 0 | 5 |
| | | Hit Rate | 100.0% |
| 0.5 Threshold (19/30) | | Select | Non Select |
| | Select | 19 | 0 |
| | Non Select | 0 | 11 |
| | | Hit Rate | 100.0% |
| 0.6 Threshold (7/30) | | Select | Non Select |
| | Select | 7 | 0 |
| | Non Select | 1 | 22 |
| | | Hit Rate | 96.7% |
| JIEDDO Selection | | Select | Non Select |
| | Select | 14 | 1 |
| | Non Select | 1 | 14 |
| | | Hit Rate | 93.3% |

| JIEDDO DATA ±50% Bound | | | |
|---|---|---|---|
| 0.4 Threshold (25/30) | | Select | Non Select |
| | Select | 25 | 0 |
| | Non Select | 0 | 5 |
| | | Hit Rate | 100.0% |
| 0.5 Threshold (19/30) | | Select | Non Select |
| | Select | 19 | 0 |
| | Non Select | 0 | 11 |
| | | Hit Rate | 100.0% |
| 0.6 Threshold (7/30) | | Select | Non Select |
| | Select | 7 | 0 |
| | Non Select | 1 | 22 |
| | | Hit Rate | 96.7% |
| JIEDDO Selection | | Select | Non Select |
| | Select | 14 | 1 |
| | Non Select | 1 | 14 |
| | | Hit Rate | 93.3% |

| JIEDDO DATA 0 to 1 Bound | | | |
|---|---|---|---|
| 0.4 Threshold (25/30) | | Select | Non Select |
| | Select | 24 | 1 |
| | Non Select | 0 | 5 |
| | | Hit Rate | 96.7% |
| 0.5 Threshold (19/30) | | Select | Non Select |
| | Select | 19 | 0 |
| | Non Select | 0 | 11 |
| | | Hit Rate | 100.0% |
| 0.6 Threshold (7/30) | | Select | Non Select |
| | Select | 7 | 0 |
| | Non Select | 1 | 22 |
| | | Hit Rate | 96.7% |
| JIEDDO Selection | | Select | Non Select |
| | Select | 14 | 1 |
| | Non Select | 1 | 14 |
| | | Hit Rate | 93.3% |

Comparing the mean of the selected group to the mean of the non-selected group and the variance of the two groups yields similar results as the MAJCOM data. Table 8 shows a portion of this data with the full results in Appendix E.

Table 8 JIEDDO Sample of Means and Variances

| 0.4 Selection Cutoff | Measure 1 | Measure 2 | Measure 3 | Measure 4 | Measure 5 | Measure 6 |
|---|---|---|---|---|---|---|
| Selected Mean | 0.530 | 0.510 | 0.730 | 0.540 | 0.310 | 0.750 |
| Non-selected Mean | 0.550 | 0.000 | 0.750 | 0.452 | 0.250 | 0.700 |
| Selected Variance | 0.064 | 0.166 | 0.005 | 0.193 | 0.048 | 0.068 |
| Non-selected Variance | 0.013 | 0.000 | 0.000 | 0.254 | 0.031 | 0.044 |

| 0.5 Selection Cutoff | Measure 1 | Measure 2 | Measure 3 | Measure 4 | Measure 5 | Measure 6 |
|---|---|---|---|---|---|---|
| Selected Mean | 0.553 | 0.549 | 0.724 | 0.684 | 0.250 | 0.789 |
| Non-selected Mean | 0.500 | 0.212 | 0.750 | 0.251 | 0.386 | 0.659 |
| Selected Variance | 0.039 | 0.174 | 0.006 | 0.163 | 0.035 | 0.043 |
| Non-selected Variance | 0.088 | 0.115 | 0.000 | 0.142 | 0.055 | 0.091 |

| | | | |
|---|---|---|---|
| (green) | Denotes a weighted attribute with 0 to 1 | (yellow) | Denotes attribute with zero weight assigned |

As previously shown, when the bounds for the weights are relaxed to 0 to 1, weight is assigned to those measures that typically have the greatest difference between selected and non-selected means. There were some instances where there was a significant difference in means but a zero weight was still assigned. This usually occurred when the variances of the two groups was large enough that it reduced the effectiveness of the differences in the means to discriminate.

## IV.D Random Data

In order to try to see how the math programming and discriminant analysis would respond to different alternative sets, a random set of 42 alternatives was created using random values between 0 and 1 for 14 attribute measures. The random values for the attributes were created using the RAND function in Microsoft Excel. The original weights of the MAJCOM data were used as the starting weight for the model and value score cutoff thresholds of 0.4, 0.5 and 0.6 were applied to determine the selection groups. Table 9 shows the resulting weights obtained when allowed to vary between 0 and 1.

Table 9 Random Alternative Set Weights for 0 to 1 Bounds

| Selection | Attribute Weights | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Threshold (# Sel / | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 0.4 (31/42) | 0.202 | 0.124 | 0 | 0.585 | 0.043 | 0 | 0 | 0 | 0 | 0 | 0 | 0.036 | 0.01 | 0 |
| 0.5 (19/42) | 0 | 0 | 0 | 0.316 | 0.38 | 0 | 0 | 0 | 0 | 0 | 0 | 0.007 | 0.297 | 0 |
| Original Weights | 0.06 | 0.17 | 0.08 | 0.17 | 0.17 | 0.012 | 0.03 | 0.018 | 0.06 | 0.06578 | 0.08211 | 0.03289 | 0.02461 | 0.02461 |

The random data had comparable hit rates to the previous examples as shown in Table 10 below. The high hit rates reflect the fact that discriminant function does a good job of classify the alternatives into selected and non-selected groups.

Table 10 Randon Alternative Set Confusion Matrix

| Random DATA 0 to 1 Bound | | | |
|---|---|---|---|
| 0.4 Threshold (31/42) | | Select | Non Select |
| | Select | 31 | 0 |
| | Non Select | 0 | 11 |
| | | Hit Rate | 100.0% |
| 0.5 Threshold (19/42) | | Select | Non Select |
| | Select | 19 | 0 |
| | Non Select | 1 | 22 |
| | | Hit Rate | 97.6% |

Comparing the mean of the selected group to the mean of the non-selected group and the variance of the two groups yields similar results as the MAJCOM and JIEDDO data.

## IV.E Specific Data Set

Finally, a very specific data set was created to help in the understanding of which attributes were getting zero weight and which were getting a high proportion of the global weight. The alternative set shown in Table 11 was specifically developed in an attempt to analyze a low dimensional problem to see how weight was being applied.

Table 11 Specific Alternative Data

| | Measure 1 | Measure 2 | Measure 3 |
|---|---|---|---|
| Alt 1 | 0.5 | 0.6 | 0.7 |
| Alt 2 | 0.7 | 0.6 | 0.5 |
| Alt 3 | 0.7 | 0.1 | 0.7 |
| Alt 4 | 0.5 | 0.2 | 0.5 |
| Alt 5 | 0.4 | 0.6 | 0.4 |

The process was run with an original weighting of 0.333 for each attribute measure and a value score cutoff threshold of 0.6. From the selection threshold, Alt 1 and Alt 2 belong to the selected group and Alts 3 through 5 are in the non-selected group. The resultant weighting from this discriminant analysis and compromise programming yields weights of 0.003 for Measure 1, 0.994 for Measure 2 and 0.003 for Measure 3.

The process was applied a second time with an original weighting of 0.4 for Measure 1, 0.2 for Measure 2 and 0.4 for Measure 3. The selection threshold was again set to 0.6. This criteria assigned Alt 1, Alt 2 and Alt 3 to the selected group and Alts 4 and 5 to the non-selected group. The resultant weighting from this discriminant analysis and compromise programming yields weights of 0.5 for Measure 1, 0 for Measure 2 and 0.5 for Measure 3.  The classification hit rate for both runs was 100%.

### IV.F Example of Inconsistent Decision

In order to create an inconsistent decision, the MAJCOM data was used and 25 alternatives were chosen to be in the selected group, leaving 17 in the non-selected group. The alternatives were distributed such that there were alternatives with high value and low value scores in both the selected and non-selected sets.  The confusion matrix in Table 12 shows the impact of this inconsistent behavior.

Table 12 Inconsistent Decision Confusion Matrix

| | Select | Non Select |
|---|---|---|
| Select | 20 | 5 |
| Non Select | 7 | 10 |
| | Hit Rate | 71.43% |

This poor hit rate demonstrates that the decision made was inconsistent with the information captured by the alternative attribute data.   A decision such as this could illustrate that the VFT model is missing some attribute or information that is important to the decision maker or that there are higher order effects, non-linear effects, or interactions that could not be accurately captured by a linear discriminant model.

### IV.G Summary

This chapter provided a clear and concise application of the methodologies presented in chapter 3 to several sets of alternative data and VFT models.  We demonstrated an ability to generate an optimized discriminate function that can correctly classify alternatives into selected and non-selected groups and provide insight into which attribute measures are actually the most discriminating for the given set of alternatives. Given a new alternative set for any of the given VFT models we can accurately predict if it would be assigned to the selected or non-selected groups.

# V. Conclusions and Recommendations

## V.A Introduction

Air Force decision makers face difficult decisions that can have significant impact to future force capabilities and structure. Often these decisions are made more difficult by the fact that the goals of the decision are often conflicting. VFT models provide a tool that allows analysts to help the DM make tradeoffs between conflicting goals. A DM needs to have a detailed, robust and mathematically rigorous model to aid in their decision making. These models should be validated in order to ensure that the mathematical rigor is sound and accurate. Also, any additional insight provided to the DM can better prepare them for the next time the VFT model may be used.

## V.B Research Contributions

The goal of this research was to contribute to the fields of Decision Analysis, specifically Value Focused Thinking, by providing a technique to validate models and provide insights to alternative sets. This research combined Discriminant Analysis and Compromise Programming techniques to provide insights into the decision made.

The first contribution is in the area of VFT model validation. This research applied a two group discriminant analysis technique to statistically generate a classification function that can accurately predict assign of alternatives based upon any VFT model and alternative set provided.

The second contribution is in the area of tradespace or sensitivity analysis. By allowing the discriminant function and compromise programming to find a set of weights with a 0 to 1 bound, we can see what attribute measures are the discriminating factors.

The measures with the most statistical weight assigned have the most discriminatory power for a given alternative set which may or may not correspond the original VFT weights. If an attribute measure was assigned little or no statistical weight it means that either most of the alternatives scored the same, or the means of the selected and non-selected groups were not statistically different enough to contribute to discriminate using that attribute measure. Another technique to provide insight into alternative sets provided by Dees et al transforms the value functions to amplify the differences between alternatives (Dees et al, 2010).

### V.C Recommendations for Further Research

The methodology presented in this research applied linear discriminant functions to validate the VFT models. Future Research could apply other multivariate techniques such as higher order functions or neural networks if a linear discriminant model indicates inconsistent behavior. This could account for any interactions between attributes, higher order or non-linear effects.

### V.D Conclusions

Decision Analysis and VFT models provide robust, reproducible tools to DMs and analysts to allow complex decisions to be represented in an understandable way. VFT models are widely used in the Air Force and should be validated as often as feasible. This research demonstrated a sound statistical method using discriminant analysis and compromise programming to valid and provides insights into VFT models and alternative sets.

## Appendix A: MATLAB Code Example

```
clc
clear all;
close all;

%# Tenets   Primary Gap Classification  Months Useful   Performance
%Suitability    Interop. Issues TRL     Months to Fielding  % Max
Capacity  Interaction Min/Hr  Training Hours
ALT=[0.5    1   0.75    1   0   1   1   0.9 0.7 1   1   1
0.75    0.22    0.75    1   0.5 1   1   0.9 0.6 0.912   1   0.701
0.75    0.78    0.75    1   0.25    0.75    0.5 0.9 0.6 0.871   0.233
0.918
1   0.78    0.75    1   0.25    0.5 0.5 0.93    0.767   0.912   0
0.163
0.5 0.22    0.75    1   0.25    0.75    0.5 0.15    0.867   0.912   1
0.701
0.5 1   0.75    0.187   0.5 0.5 0.5 0.9 0.7 0.912   0.039   0.106
0.5 1   0.75    0.5 0.25    0.5 0.5 0.2 0.475   0.955   0.039   0.596
0.75    0.33    0.5 0.023   0.25    1   1   0.9 0.8 0.912   0.617
0.701
0.5 1   0.75    0.187   0.5 0.75    0.5 0.3 0.6 0.795   0.039   0.5
0.5 1   0.75    0.5 0.25    0.5 0.5 0.9 0.6 0.795   0   0.163
0.5 0   0.75    1   0.5 0.75    0.5 0.2 0.475   0.871   0.617   0.843
0.5 1   0.75    0.5 0.25    0.5 0.5 0.15    0.733   0.795   0   0.241
0.5 1   0.75    0.023   0.25    0.75    0.5 0.3 0.733   0.795   0
0.701
0.5 0   0.75    0.074   0.5 1   0.5 0.9 0.867   0.795   1   0.918
0.5 0   0.75    0.187   0.5 1   0.5 0.3 0.475   0.396   0.039   0.701
0.5 0.44    0.75    1   0   1   1   0.3 0.35    1   1   1
0.5 0.44    0.75    1   0   1   1   0   0.35    1   1   1
0.5 0   0.5 1   0   1   1   0   0.7 1   1   1
0   0   0.75    1   0   1   1   0.2 0.558   1   1   1
0.75    0.22    0.75    1   0.25    0.75    0.5 0.9 0.8 0.631   0.617
0
0   0.22    0.75    0.074   0.5 1   0.5 0.9 0.475   0.955   0.617
0.701
0.5 0   0.75    0.187   0.75    0.5 0   0.93    0.867   0.955   0.617
0.918
0.75    0.89    0.75    0.187   0.25    0.75    0   0.2 0.433   0.912
0.233   0
0.5 0.44    0.75    0.023   0.5 1   0   0.2 0.475   0.396   1   1
1   0.78    0.75    0.004   0.25    0.5 0   0.1 0.558   0.955   0.039
0.5
0   0   0.75    0.023   0.75    0   1   0.15    0.933   0.016   1
0.701
0.75    0   0.75    1   0.25    0.75    0   0.1 0.35    0.396   0.617
0
0.5 0   0.75    0.074   0.25    0.5 0.5 0.1 0.475   0.396   1   0.701
0.5 0   0.75    1   0.25    0.5 0   0   0.35    0.795   0.039   0.5
0.5 0   0.75    0.001   0   0.75    0   0   0.8 0   0.039   0];
Weights=[0.056  0.176   0.056   0.112   0.11    0.056   0.091   0.037
0.056   0.087   0.1 0.05];

full=1
```

```
if full == 1

Scores=ALT*Weights';
pop1=ALT(1:15,1:12);
pop2=ALT(16:30,1:12);
covp=(1/28)*((14*cov(pop1))+(14*cov(pop2)))
b=inv(covp)*(mean(pop1)'-mean(pop2)')
dscores=ALT(:,1:12)*b
mid=.5*(mean(pop1)'-mean(pop2)')'*inv(covp)*(mean(pop1)'+mean(pop2)')
tp=0;fn=0;tn=0;fp=0;
for i=1:30
    if dscores(i) >= mid & i <= 15
        tp=tp+1
    elseif dscores(i)<= mid & i <= 15
        fn=fn+1
    elseif dscores(i) <=  mid & i > 15
        tn=tn+1
    else
        fp=fp+1
    end
end
CA=(tp+tn)/(tp+tn+fn+fp)
E=inv(covp)
M1=mean(pop1)
M2=mean(pop2)
pop=[pop1;pop2]
for i=1:30
    num=exp((-.5)*(pop(i,:)-M1)*E*(pop(i,:)-M1)');
    denom=num+exp((-.5)*(pop(i,:)-M2)*E*(pop(i,:)-M2)')
    pp(i)=num/denom
end

global pop pp

p=12
x0=ones(1,p)*(1/p);
lb=Weights*0.1;
ub=Weights*1.1;
Aeq=ones(1,p);
beq=1;

[x,fval,exitflag]=fmincon(@globalfun,x0,[],[],Aeq,beq,lb,ub)

pred=pop*x'

else

Scores=ALT*Weights';

pop1=ALT(1:15,1:12);
pop2=ALT(16:30,1:12);
covp=(1/28)*((14*cov(pop1))+(14*cov(pop2)))
b=inv(covp)*(mean(pop1)'-mean(pop2)')
dscores=ALT(:,1:12)*b
```

```
mid=.5*(mean(pop1)'-mean(pop2)')'*inv(covp)*(mean(pop1)'+mean(pop2)')
tp=0;fn=0;tn=0;fp=0;
for i=1:30
    if dscores(i) >= mid & i <= 15
        tp=tp+1
    elseif dscores(i)<= mid & i <= 15
        fn=fn+1
    elseif dscores(i) <=  mid & i > 15
        tn=tn+1
    else
        fp=fp+1
    end
end
CA=(tp+tn)/(tp+tn+fn+fp)
E=inv(covp)
M1=mean(pop1)
M2=mean(pop2)
pop=[pop1;pop2]
for i=1:30
    num=exp((-.5)*(pop(i,:)-M1)*E*(pop(i,:)-M1)');
    denom=num+exp((-.5)*(pop(i,:)-M2)*E*(pop(i,:)-M2)')
    pp(i)=num/denom
end

end
```

# Appendix B: MAJCOM Alternative Set

Table 13 MAJCOM Alternative Set

| | Measure 1 | Measure 2 | Measure 3 | Measure 4 | Measure 5 | Measure 6 | Measure 7 | Measure 8 | Measure 9 | Measure 10 | Measure 11 | Measure 12 | Measure 13 | Measure 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Alternative 1 | 1 | 1 | 1 | 0.994599525 | 0.75 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0.66 |
| Alternative 2 | 1 | 1 | 1 | 1 | 0.9 | 1 | 1 | 0 | 1 | 1 | 0.8 | 0.6 | 1 | 0.66 |
| Alternative 3 | 1 | 1 | 1 | 1 | 0.65 | 0 | 1 | 1 | 1 | 1 | 0.8 | 0.8 | 0.666666667 | 0.66 |
| Alternative 4 | 1 | 0.75 | 1 | 1 | 0.9 | 1 | 1 | 0 | 1 | 1 | 0.5 | 0.6 | 1 | 1 |
| Alternative 5 | 0.75 | 0.75 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0.666666667 | 0.66 |
| Alternative 6 | 1 | 1 | 0.5 | 0.994599525 | 0.65 | 0 | 1 | 1 | 1 | 1 | 0.8 | 0.8 | 0.666666667 | 0.66 |
| Alternative 7 | 1 | 1 | 0.5 | 0.985918162 | 0.65 | 0 | 1 | 1 | 1 | 1 | 0.8 | 0.8 | 0.666666667 | 0.66 |
| Alternative 8 | 1 | 1 | 0.5 | 0.97196271 | 0.65 | 0 | 1 | 0 | 1 | 1 | 0.8 | 0.8 | 0.666666667 | 0.66 |
| Alternative 9 | 1 | 0.7 | 1 | 0.612501618 | 1 | 1 | 1 | 0.75 | 1 | 1 | 0.8 | 0.4 | 0.666666667 | 0.66 |
| Alternative 10 | 1 | 0.75 | 1 | 0.949529063 | 0.65 | 0 | 1 | 1 | 1 | 1 | 0.8 | 0.4 | 0.666666667 | 0.66 |
| Alternative 11 | 1 | 1 | 0.5 | 0.913466561 | 0.65 | 0 | 1 | 1 | 1 | 0.666666667 | 0.8 | 0.8 | 0.666666667 | 1 |
| Alternative 12 | 1 | 0.7 | 0.1 | 0.97196271 | 0.9 | 1 | 1 | 0 | 1 | 1 | 0.8 | 0.8 | 1 | 0.66 |
| Alternative 13 | 1 | 1 | 0 | 1 | 0.9 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0.66 |
| Alternative 14 | 1 | 0.7 | 1 | 1 | 0.9 | 1 | 0 | 0 | 1 | 1 | 0.2 | 0.6 | 0.333333333 | 0.66 |
| Alternative 15 | 1 | 1 | 0.3 | 0.27957697 | 0.9 | 1 | 1 | 0.75 | 1 | 1 | 1 | 0 | 0.666666667 | 1 |
| Alternative 16 | 1 | 1 | 1 | 0.18674946 | 0.9 | 1 | 1 | 0.75 | 1 | 0.666666667 | 0.8 | 0 | 0.666666667 | 0.66 |
| Alternative 17 | 1 | 1 | 1 | 0.949529063 | 0.5 | 0 | 1 | 0.75 | 0 | 0.666666667 | 1 | 0 | 0.666666667 | 0.33 |
| Alternative 18 | 1 | 0.75 | 0 | 0.985918162 | 0.9 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0.666666667 | 0.66 |
| Alternative 19 | 1 | 0.7 | 1 | 1 | 0.7 | 0 | 0 | 0 | 0 | 1 | 0.8 | 1 | 1 | 0.33 |
| Alternative 20 | 1 | 0.7 | 0 | 0.855495422 | 0.9 | 1 | 1 | 0.75 | 1 | 1 | 0.5 | 0 | 0.666666667 | 0.66 |
| Alternative 21 | 1 | 1 | 0 | 0.994599525 | 0.75 | 1 | 0 | 0.75 | 1 | 0.666666667 | 0.5 | 0 | 0.666666667 | 0.66 |
| Alternative 22 | 0.5 | 0.75 | 1 | 0.994599525 | 0.7 | 0 | 0 | 1 | 0 | 1 | 0.8 | 0.4 | 0.666666667 | 0.66 |
| Alternative 23 | 1 | 0.75 | 0 | 1 | 0.9 | 0 | 0 | 0.75 | 0 | 1 | 0.8 | 0.6 | 1 | 0.66 |
| Alternative 24 | 1 | 0.75 | 1 | 0.123269143 | 0.9 | 1 | 1 | 0.75 | 1 | 0.666666667 | 1 | 0 | 0.333333333 | 1 |
| Alternative 25 | 1 | 0.75 | 0.3 | 0.415318992 | 1 | 1 | 1 | 0.75 | 0 | 1 | 1 | 0.4 | 0.666666667 | 0.66 |
| Alternative 26 | 1 | 0.75 | 1 | 0.985918162 | 0.9 | 1 | 1 | 0 | 0 | 0.333333333 | 0.1 | 0 | 0.333333333 | 1 |
| Alternative 27 | 0.75 | 0.75 | 1 | 0.612501618 | 0.65 | 0 | 1 | 0 | 0 | 1 | 0.8 | 0.6 | 1 | 0.66 |
| Alternative 28 | 1 | 1 | 0 | 0.762305745 | 0.5 | 0 | 1 | 0.75 | 0 | 1 | 0.8 | 0.6 | 1 | 0.66 |
| Alternative 29 | 0.75 | 0.7 | 0.3 | 0.612501618 | 0.65 | 0 | 1 | 0 | 1 | 1 | 0.8 | 0.4 | 0.666666667 | 1 |
| Alternative 30 | 0.75 | 0.7 | 0.3 | 0.762305745 | 0.5 | 0 | 1 | 0.75 | 1 | 0.666666667 | 1 | 0 | 0.666666667 | 0.66 |
| Alternative 31 | 1 | 0.7 | 0 | 0.006492522 | 0.9 | 1 | 1 | 0.75 | 0 | 0.666666667 | 1 | 1 | 1 | 1 |
| Alternative 32 | 1 | 1 | 0 | 0.27957697 | 0.5 | 0 | 1 | 0 | 0 | 1 | 0.8 | 1 | 0.666666667 | 0.66 |
| Alternative 33 | 1 | 0.7 | 0.5 | 0.415318992 | 0.9 | 1 | 0 | 0 | 1 | 0.333333333 | 0.1 | 0 | 0.333333333 | 1 |
| Alternative 34 | 0.75 | 0.7 | 0.5 | 0.18674946 | 0.75 | 0 | 0 | 0 | 1 | 0.666666667 | 0.8 | 0.6 | 0.666666667 | 0.33 |
| Alternative 35 | 0.5 | 0.75 | 0 | 0.913466561 | 0.5 | 1 | 0 | 0 | 0 | 1 | 0.5 | 0.6 | 0.666666667 | 0.66 |
| Alternative 36 | 0.75 | 0.75 | 0.5 | 0.015986561 | 0.7 | 1 | 0 | 0 | 0 | 1 | 0.8 | 1 | 1 | 0.66 |
| Alternative 37 | 1 | 0.75 | 0 | 0.02986973 | 0.75 | 0 | 1 | 0 | 0 | 1 | 0.8 | 0.8 | 1 | 0.33 |
| Alternative 38 | 1 | 1 | 0 | 0.079857975 | 0.7 | 0 | 1 | 0 | 0 | 0.666666667 | 0.8 | 0 | 0.666666667 | 0.66 |
| Alternative 39 | 1 | 1 | 0 | 0.123269143 | 0.5 | 0 | 1 | 1 | 0 | 0.333333333 | 0.8 | 0.6 | 0.666666667 | 1 |
| Alternative 40 | 0.75 | 0.7 | 0 | 0.050171142 | 0.7 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| Alternative 41 | 1 | 0.7 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0.333333333 | 0 | 0 | 0.333333333 | 1 |
| Alternative 42 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0.8 | 0 | 0.333333333 | 0.66 |

# Appendix C: MAJCOM Means and Variances Table

Table 14 MAJCOM Means and Variances Table

| 0.5 Selection Cutoff | Measure 1 | Measure 2 | Measure 3 | Measure 4 | Measure 5 | Measure 6 | Measure 7 | Measure 8 | Measure 9 | Measure 10 | Measure 11 | Measure 12 | Measure 13 | Measure 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Selected Mean | 0.931 | 0.836 | 0.495 | 0.675 | 0.758 | 0.450 | 0.775 | 0.450 | 0.550 | 0.875 | 0.775 | 0.450 | 0.742 | 0.712 |
| Non-selected Mean | 0.500 | 0.350 | 0.000 | 1.000 | 0.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.667 | 0.400 | 0.000 | 0.333 | 0.830 |
| Selected Variance | 0.019 | 0.019 | 0.188 | 0.144 | 0.025 | 0.254 | 0.179 | 0.196 | 0.254 | 0.044 | 0.054 | 0.139 | 0.043 | 0.038 |
| Non-selected Variance | 0.500 | 0.245 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.222 | 0.320 | 0.000 | 0.000 | 0.058 |

| 0.6 Selection Cutoff | Measure 1 | Measure 2 | Measure 3 | Measure 4 | Measure 5 | Measure 6 | Measure 7 | Measure 8 | Measure 9 | Measure 10 | Measure 11 | Measure 12 | Measure 13 | Measure 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Selected Mean | 0.950 | 0.847 | 0.610 | 0.831 | 0.780 | 0.467 | 0.833 | 0.542 | 0.667 | 0.911 | 0.787 | 0.413 | 0.733 | 0.706 |
| Non-selected Mean | 0.813 | 0.729 | 0.125 | 0.342 | 0.575 | 0.500 | 0.667 | 0.146 | 0.167 | 0.750 | 0.683 | 0.467 | 0.694 | 0.747 |
| Selected Variance | 0.015 | 0.019 | 0.180 | 0.070 | 0.024 | 0.257 | 0.144 | 0.186 | 0.230 | 0.030 | 0.051 | 0.127 | 0.041 | 0.029 |
| Non-selected Variance | 0.092 | 0.069 | 0.051 | 0.159 | 0.091 | 0.273 | 0.242 | 0.119 | 0.152 | 0.083 | 0.103 | 0.192 | 0.070 | 0.064 |

| 0.7 Selection Cutoff | Measure 1 | Measure 2 | Measure 3 | Measure 4 | Measure 5 | Measure 6 | Measure 7 | Measure 8 | Measure 9 | Measure 10 | Measure 11 | Measure 12 | Measure 13 | Measure 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Selected Mean | 0.970 | 0.860 | 0.628 | 0.847 | 0.808 | 0.520 | 0.800 | 0.590 | 0.720 | 0.933 | 0.804 | 0.432 | 0.733 | 0.688 |
| Non-selected Mean | 0.824 | 0.744 | 0.241 | 0.461 | 0.594 | 0.412 | 0.765 | 0.191 | 0.235 | 0.765 | 0.688 | 0.424 | 0.706 | 0.761 |
| Selected Variance | 0.012 | 0.019 | 0.180 | 0.079 | 0.020 | 0.260 | 0.167 | 0.182 | 0.210 | 0.019 | 0.040 | 0.136 | 0.037 | 0.028 |
| Non-selected Variance | 0.068 | 0.052 | 0.121 | 0.151 | 0.070 | 0.257 | 0.191 | 0.129 | 0.191 | 0.080 | 0.101 | 0.159 | 0.068 | 0.053 |

| 0.8 Selection Cutoff | Measure 1 | Measure 2 | Measure 3 | Measure 4 | Measure 5 | Measure 6 | Measure 7 | Measure 8 | Measure 9 | Measure 10 | Measure 11 | Measure 12 | Measure 13 | Measure 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Selected Mean | 0.979 | 0.888 | 0.758 | 0.950 | 0.779 | 0.417 | 1.000 | 0.479 | 1.000 | 0.972 | 0.808 | 0.650 | 0.778 | 0.717 |
| Non-selected Mean | 0.883 | 0.783 | 0.357 | 0.587 | 0.698 | 0.500 | 0.700 | 0.408 | 0.333 | 0.822 | 0.737 | 0.340 | 0.700 | 0.718 |
| Selected Variance | 0.005 | 0.020 | 0.101 | 0.012 | 0.022 | 0.265 | 0.000 | 0.255 | 0.000 | 0.009 | 0.015 | 0.074 | 0.027 | 0.018 |
| Non-selected Variance | 0.051 | 0.039 | 0.183 | 0.157 | 0.061 | 0.259 | 0.217 | 0.179 | 0.230 | 0.059 | 0.086 | 0.144 | 0.056 | 0.047 |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Column Variance | 0.039 | 0.035 | 0.190 | 0.142 | 0.050 | 0.256 | 0.172 | 0.196 | 0.256 | 0.049 | 0.066 | 0.142 | 0.048 | 0.038 |
| Column Mean | 0.911 | 0.813 | 0.471 | 0.691 | 0.721 | 0.476 | 0.786 | 0.429 | 0.524 | 0.865 | 0.757 | 0.429 | 0.722 | 0.718 |

| | |
|---|---|
| (blue) | Denotes a weighted attribute with 0 to 1 bound |
| (pink) | Denotes attribute with zero weight assigned |

# Appendix D: JIEDDO Alternative Set

Table 15 JIEDDO Alternative Set

| Alternative | # Tenets | Primary Gap | Classification | Months Useful | Performance | Suitability | Interop. Issues | TRL | Months to Fielding | % Max Capacity | Interaction Min/Hr | Training Hours | Training Level | Value |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| DD | 0.5 | 1 | 0.75 | 1 | 0 | 1 | 1 | 0.9 | 0.7 | 1 | 1 | 1 | 0.6 | 0.8223 |
| BB | 0.75 | 0.22 | 0.75 | 1 | 0.5 | 1 | 1 | 0.9 | 0.6 | 0.912 | 1 | 0.701 | 0.6 | 0.72467 |
| F | 0.5 | 0.44 | 0.75 | 1 | 0 | 1 | 1 | 0.3 | 0.35 | 1 | 1 | 1 | 0.6 | 0.68194 |
| CC | 0.75 | 0.78 | 0.75 | 1 | 0.25 | 0.75 | 0.5 | 0.9 | 0.6 | 0.871 | 0.233 | 0.918 | 0.6 | 0.676251 |
| E | 0.5 | 0.44 | 0.75 | 1 | 0 | 1 | 1 | 0 | 0.35 | 1 | 1 | 1 | 0.6 | 0.67084 |
| AA | 1 | 0.78 | 0.75 | 1 | 0.25 | 0.5 | 0.5 | 0.93 | 0.767 | 0.912 | 0 | 0.163 | 0.6 | 0.632792 |
| Z | 0.5 | 0.22 | 0.75 | 1 | 0.25 | 0.75 | 0.5 | 0.15 | 0.867 | 0.912 | 1 | 0.701 | 0.6 | 0.610872 |
| J | 0.5 | 0 | 0.5 | 1 | 0 | 1 | 1 | 0 | 0.7 | 1 | 1 | 1 | 0.6 | 0.599 |
| B | 0 | 0 | 0.75 | 1 | 0 | 1 | 1 | 0.2 | 0.558 | 1 | 1 | 1 | 0.6 | 0.584448 |
| R | 0.5 | 1 | 0.75 | 0.187 | 0.5 | 0.5 | 0.5 | 0.9 | 0.7 | 0.912 | 0.039 | 0.106 | 0.6 | 0.575637 |
| T | 0.5 | 1 | 0.75 | 0.5 | 0.25 | 0.5 | 0.5 | 0.2 | 0.475 | 0.955 | 0.039 | 0.596 | 0.6 | 0.573493 |
| X | 0.75 | 0.33 | 0.5 | 0.023 | 0.25 | 1 | 1 | 0.9 | 0.8 | 0.912 | 0.617 | 0.701 | 0.6 | 0.570985 |
| Y | 0.5 | 1 | 0.75 | 0.187 | 0.5 | 0.75 | 0.5 | 0.3 | 0.6 | 0.795 | 0.039 | 0.5 | 0.6 | 0.569837 |
| S | 0.5 | 1 | 0.75 | 0.5 | 0.25 | 0.5 | 0.5 | 0.9 | 0.6 | 0.795 | 0 | 0.163 | 0.6 | 0.56535 |
| P | 0.75 | 0.22 | 0.75 | 1 | 0.25 | 0.75 | 0.5 | 0.9 | 0.8 | 0.631 | 0.617 | 0 | 0.6 | 0.552399 |
| W | 0.5 | 0 | 0.75 | 1 | 0.5 | 0.75 | 0.5 | 0.2 | 0.475 | 0.871 | 0.617 | 0.843 | 0.6 | 0.549229 |
| D | 0.5 | 1 | 0.75 | 0.5 | 0.25 | 0.5 | 0.5 | 0.15 | 0.733 | 0.795 | 0 | 0.241 | 0.6 | 0.548948 |
| C | 0.5 | 1 | 0.75 | 0.023 | 0.25 | 0.75 | 0.5 | 0.3 | 0.733 | 0.795 | 0 | 0.701 | 0.6 | 0.538074 |
| U | 0.5 | 0 | 0.75 | 0.074 | 0.5 | 1 | 0.5 | 0.9 | 0.867 | 0.795 | 1 | 0.918 | 0.6 | 0.53684 |
| L | 0 | 0.22 | 0.75 | 0.074 | 0.5 | 1 | 0.5 | 0.9 | 0.475 | 0.955 | 0.617 | 0.701 | 0.6 | 0.497437 |
| I | 0.5 | 0 | 0.75 | 0.187 | 0.75 | 0.5 | 0 | 0.93 | 0.867 | 0.955 | 0.617 | 0.918 | 0.6 | 0.487285 |
| Q | 0.75 | 0.89 | 0.75 | 0.187 | 0.25 | 0.75 | 0 | 0.2 | 0.433 | 0.912 | 0.233 | 0 | 0.6 | 0.482003 |
| G | 0.5 | 0.44 | 0.75 | 0.023 | 0.5 | 1 | 0 | 0.2 | 0.475 | 0.396 | 1 | 1 | 0.6 | 0.479416 |
| O | 1 | 0.78 | 0.75 | 0.004 | 0.25 | 0.5 | 0 | 0.1 | 0.558 | 0.955 | 0.039 | 0.5 | 0.6 | 0.457869 |
| N | 0 | 0 | 0.75 | 0.023 | 0.75 | 0 | 1 | 0.15 | 0.933 | 0.016 | 1 | 0.701 | 0.6 | 0.407324 |
| A | 0.75 | 0 | 0.75 | 1 | 0.25 | 0.75 | 0 | 0.1 | 0.35 | 0.396 | 0.617 | 0 | 0.6 | 0.389879 |
| K | 0.5 | 0 | 0.75 | 0.074 | 0.25 | 0.5 | 0.5 | 0.1 | 0.475 | 0.396 | 1 | 0.701 | 0.6 | 0.379038 |
| M | 0.5 | 0 | 0.75 | 1 | 0.25 | 0.5 | 0 | 0 | 0.35 | 0.795 | 0.039 | 0.5 | 0.6 | 0.372793 |
| V | 0.5 | 0 | 0.75 | 0.187 | 0.5 | 1 | 0.5 | 0.3 | 0.475 | 0.396 | 0.039 | 0.701 | 0.6 | 0.370987 |
| H | 0.5 | 0 | 0.75 | 0.001 | 0 | 0.75 | 0 | 0 | 0.8 | 0 | 0.039 | 0 | 0.6 | 0.168105 |

# Appendix E: JIEDDO Means and Variances Table

Table 16 JIEDDO Means and Variances Table

| 0.4 Selection Cutoff | Measure 1 | Measure 2 | Measure 3 | Measure 4 | Measure 5 | Measure 6 | Measure 7 | Measure 8 | Measure 9 | Measure 10 | Measure 11 | Measure 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Selected Mean | 0.530 | 0.510 | 0.730 | 0.540 | 0.310 | 0.750 | 0.580 | 0.496 | 0.641 | 0.842 | 0.548 | 0.643 |
| Non-selected Mean | 0.550 | 0.000 | 0.750 | 0.452 | 0.250 | 0.700 | 0.200 | 0.100 | 0.490 | 0.397 | 0.347 | 0.380 |
| Selected Variance | 0.064 | 0.166 | 0.005 | 0.193 | 0.048 | 0.068 | 0.118 | 0.142 | 0.028 | 0.048 | 0.188 | 0.118 |
| Non-selected Variance | 0.013 | 0.000 | 0.000 | 0.254 | 0.031 | 0.044 | 0.075 | 0.015 | 0.034 | 0.079 | 0.196 | 0.127 |

| 0.5 Selection Cutoff | Measure 1 | Measure 2 | Measure 3 | Measure 4 | Measure 5 | Measure 6 | Measure 7 | Measure 8 | Measure 9 | Measure 10 | Measure 11 | Measure 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Selected Mean | 0.553 | 0.549 | 0.724 | 0.684 | 0.250 | 0.789 | 0.684 | 0.523 | 0.646 | 0.888 | 0.537 | 0.645 |
| Non-selected Mean | 0.500 | 0.212 | 0.750 | 0.251 | 0.386 | 0.659 | 0.227 | 0.271 | 0.563 | 0.561 | 0.476 | 0.520 |
| Selected Variance | 0.039 | 0.174 | 0.006 | 0.163 | 0.035 | 0.043 | 0.061 | 0.144 | 0.024 | 0.010 | 0.207 | 0.122 |
| Non-selected Variance | 0.088 | 0.115 | 0.000 | 0.142 | 0.055 | 0.091 | 0.118 | 0.109 | 0.042 | 0.136 | 0.171 | 0.133 |

| 0.6 Selection Cutoff | Measure 1 | Measure 2 | Measure 3 | Measure 4 | Measure 5 | Measure 6 | Measure 7 | Measure 8 | Measure 9 | Measure 10 | Measure 11 | Measure 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Selected Mean | 0.643 | 0.554 | 0.750 | 1.000 | 0.179 | 0.857 | 0.786 | 0.583 | 0.605 | 0.944 | 0.748 | 0.783 |
| Non-selected Mean | 0.500 | 0.386 | 0.728 | 0.381 | 0.337 | 0.707 | 0.435 | 0.384 | 0.619 | 0.714 | 0.444 | 0.543 |
| Selected Variance | 0.039 | 0.092 | 0.000 | 0.000 | 0.036 | 0.039 | 0.071 | 0.172 | 0.039 | 0.003 | 0.190 | 0.093 |
| Non-selected Variance | 0.057 | 0.199 | 0.005 | 0.164 | 0.043 | 0.066 | 0.121 | 0.131 | 0.030 | 0.090 | 0.174 | 0.126 |

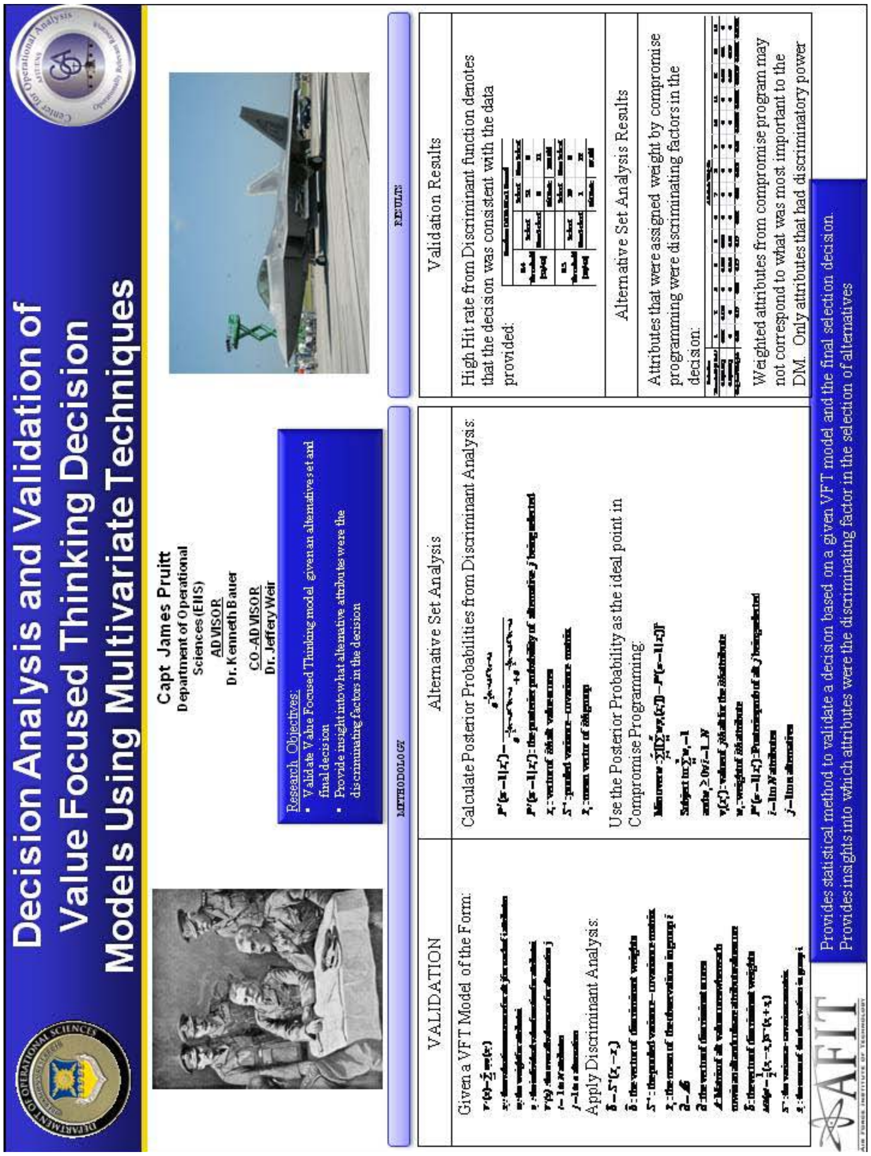
| JIEDDO Selection Cutoff | Measure 1 | Measure 2 | Measure 3 | Measure 4 | Measure 5 | Measure 6 | Measure 7 | Measure 8 | Measure 9 | Measure 10 | Measure 11 | Measure 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Selected Mean | 0.583 | 0.622 | 0.733 | 0.545 | 0.333 | 0.750 | 0.600 | 0.589 | 0.666 | 0.842 | 0.375 | 0.597 |
| Non-selected Mean | 0.483 | 0.229 | 0.733 | 0.505 | 0.267 | 0.733 | 0.433 | 0.272 | 0.565 | 0.694 | 0.655 | 0.601 |
| Selected Variance | 0.022 | 0.170 | 0.004 | 0.171 | 0.029 | 0.045 | 0.043 | 0.130 | 0.030 | 0.021 | 0.194 | 0.099 |
| Non-selected Variance | 0.099 | 0.099 | 0.005 | 0.224 | 0.064 | 0.085 | 0.183 | 0.130 | 0.040 | 0.136 | 0.157 | 0.169 |

| | | | |
|---|---|---|---|
| | Denotes a weighted attribute with 0 to 1 | | Denotes attribute with zero weight assigned |

## Appendix F: Blue Dart

The Air Force decision makers have to make billion dollar decisions every day. Analysts provide insights to the decision maker using decision analysis tools and models. Often these models that are used to make complicated decisions are not validated using mathematical or statistical techniques. This research provides a means to mathematically validate a value focused thinking decision model for a given set of alternatives that were considered. Additionally it provides insight into what aspects or attributes of the alternatives were the key factors that helped make the decision.

# Appendix G:  Quad Chart



## Bibliography


Andrews, R., Diederich, J., & Tickle, A. B. (1995). Survey and critique of techniques for extracting rules from trained artificial neural networks. *Knowledge-Based Systems, 8*(6), 373-389.

Baldi, P. F. Learning in linear neural networks: A survey. *IEEE Transactions on Neural Networks, 6*(4), 837.

Barron, F. H., & Barrett, B. E. (1996). The efficacy of SMARTER — simple multi-attribute rating technique extended to ranking. *Acta Psychologica, 93*(1-3), 23-36.

Bauer, K. W. (2010). Course Notes: OPER 685 Applied Multivariate Analysis I. Wright-Patterson Air Force Base, Ohio

Bauer, K. W. (1999). Response Surface methodology as a sensitivity analysis tool in decision analysis. *Journal of Multicriteria Decision Analysis* , pp. 162-180.

Chen, H., & Kocaoglu, D. F. (2008). A sensitivity analysis algorithm for hierarchical decision models. *European Journal of Operational Research, 185*(1), 266-288.

Clemen, Robert T (Robert Taylor), %T Making hard decisions with DecisionTools / Robert T Clemen,Terence Reilly. ([2nd rev. ] ed.)

Dees, R. A., Dabkowski, M. F., & Parnell, G. S. (2010). Decision-Focused Transformation of Additive Value Models to Improve Commmunication. *Decision Analysis* , 172-184.

Dillon, W. R., & Goldstein, M. (1984). *Multivariate Analysis: Methods and Applications.* New York: Wiley.

Erenguc, S. S., & Koehler, G. J. (1990). Survey of mathematical programming models and experimental results for linear discriminant analysis. *Managerial and Decision Economics, 11*(4), 215-225.

Ehrgott, M. (2005). *Multicriteria Optimization.* Berlin: Springer

Insua, D. R., & French, S. (1991). A framework for sensitivity analysis in discrete multi-objective decision-making. *European Journal of Operational Research, 54*(2), 176-190.

Jessop, A. (2004). Sensitivity and robustness in selection problems. *Computers & Operations Research, 31*(4), 607-622.

Keeney, R. L. (1992). *Value-Focused Thinking: a path to creative decision making.* Cambridge, Mass. Harvard University Press.

Kirkwood, C. W. *Strategic decision making : Multiobjective decision analysis with spreadsheets.* Belmont: Wadsworth

Mateos, A., Jiménez, A., & Ríos-Insua, S. (2003). Solving dominance and potential optimality in imprecise multi-attribute additive problems. *Reliability Engineering & System Safety, 79*(2), 253-262.

Moskowitz, H., Tang, J., & Lam, P. (2000). Distribution of aggregate utility using stochastic elements of additive multiattribute utility models. *Decision Sciences, 31*(2), 327-360.

Mustajoki, J., Hämäläinen, R. P., & Salo, A. (2005). Decision support by interval SMART/SWING?incorporating imprecision in the SMART and SWING methods. *Decision Sciences, 36*(2), 317-339.

Mustajoki, J., Hämäläinen, R. P., & Lindstedt, M. R. K. (2006). Using intervals for global sensitivity and worst-case analyses in multiattribute value trees. *European Journal of Operational Research, 174*(1), 278-292.

Pöyhönen, M., & Hämäläinen, R. P. (2001). On the convergence of multiattribute weighting methods. *European Journal of Operational Research, 129*(3), 569-585.

Ringuest, J. L. (1997). LP-metric sensitivity analysis for single and multi-attribute decision analysis. *European Journal of Operational Research, 98*(3), 563-570.

Skinner, D. C. *Introduction to decision analysis : A practitioner's guide to improving decision quality* (2nd ed.)

Terrin, N., Schmid, C. H., Griffith, J. L., D'Agostino, R. B., & Selker, H. P. (2003). External validity of predictive models: A comparison of logistic regression, classification trees, and neural networks. *Journal of Clinical Epidemiology, 56*(8), 721-729.

Wagner, H. M. (1995). Global sensitivity analysis. *Operations Research, 43*(6), 948-969. doi:10.1287/opre.43.6.948

Wei, Q. -., Ma, J., & Fan, Z. -. (2000). A parameter analysis method for the weight-set to satisfy preference orders of alternatives in additive multi-criteria value models. *Journal of Multi-Criteria Decision Analysis, 9*(5), 181-190.

Willy, C. J. (2009). *Robust Sensitivity Analysis for the Joint Improvised Explosive Device Defeat Organization (JIEDDO) Proposal Selection Model.* Air Force Institute of Technology.

Zhang, G. P. (2000). Neural networks for classification: A survey. *Systems, Man, and Cybernetics, Part C: Applications and Reviews, IEEE Transactions on, 30*(4), 451-462.

## Vita

James L. Pruitt graduated from South Laurel County High School in London, Kentucky. He entered the undergraduate program at Berea College, Berea, Kentucky where he graduated with a Bachelor of Arts with a major in Mathematics. He enlisted in the Air Force as a Communications, Computer, and Switching Systems Maintenance technician in 1994. His first duty station was the 325th Communications Squadron, Tyndall AFB, Florida. In spring 1998 he was assigned to the 606th Air Control Squadron, Spangdahlem AB, Germany. Then in spring of 2000 he returned to the 325th Communications Squadron at Tyndall AFB.

In March of 2004 he received his commission as a 33S from Officer Training School (OTS). His first assignment after OTS was to the Air Mobility Command (AMC) Communications Group as a Communications Manager. In 2006 he re-cored as a 61S and worked for AMC/A9 as a Mobility Operations Analyst. In the summer of 2008 he was assigned as an Operations Research Analyst to Air Force Materiel Command/A9A. In August of 2009, he entered the Graduate School of Engineering and Management, Air Force Institute of Technology. Upon graduation he will be assigned to the Defense Logistics Agency, Richmond, Virginia.

| REPORT DOCUMENTATION PAGE | | | *Form Approved* *OMB No. 074-0188* |
|---|---|---|---|



| 1. REPORT DATE *(DD-MM-YYYY)* | 2. REPORT TYPE | 3. DATES COVERED *(From – To)* |
|---|---|---|
| 24-02-2011 | **Master's Thesis** | Sep 2010- Mar 2011 |

4. TITLE AND SUBTITLE
Decision Analysis and Validation of Value Focused Thinking Decision Models Using Multivariate Analysis Techniques

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Pruitt, James, L., Captain, USAF

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(S)
Air Force Institute of Technology
Graduate School of Engineering and Management (AFIT/EN)
2950 Hobson Street, Building 642
WPAFB OH 45433-7765

8. PERFORMING ORGANIZATION REPORT NUMBER
AFIT-OR-MS-ENS-11-17

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Dr. Theodore Lewis
AFMC/A9A
4375 Chidlaw Road , Building 262
WPAFB, OH 45433
(937) 904-1139
theodore.lewis@wpafb.af.mil

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
Decision Analysis (DA) is a useful tool to assist decision makers (DM) with difficult and complex decisions using mathematical models. Value Focused Thinking (VFT) models are a useful DA tool widely employed in the Air Force. However, VFT models are rarely validated.

This research will attempt to validate any given VFT model and provide insight into the discriminating attributes of the alternative set. First, a two group discriminant analysis is applied the alternative set given the prior knowledge of the selected alternatives. Next, compromise programming is used attempt to minimize the distance between the posterior probability of an alternative being selected and its current weighted value by varying the weights. This set of optimized weights is then used in the two group discriminant analysis to classify the alternative set and attempt to validate the VFT model by selecting the same subset of alternatives chosen by the DM. Additionally, this process will provide insight into what attributes of a given alternative set are actually the discriminating factors in the decision which may or may not be the attributes that are most important to or most heavily weighted by the DM.

15. SUBJECT TERMS
Decision Analysis, Value Focused Thinking, Discriminant Analysis, Multivariate Analysis

| 16. SECURITY CLASSIFICATION OF: | | | 17. LIMITATION OF ABSTRACT | 18. NUMBER OF PAGES | 19a. NAME OF RESPONSIBLE PERSON Dr. Kenneth Bauer |
|---|---|---|---|---|---|
| a. REPORT | b. ABSTRACT | c. THIS PAGE | | | 19b. TELEPHONE NUMBER *(Include area code)* |
| **U** | **U** | **U** | **UU** | 53 | (937) 255-3636 x4328 |